B.Sc. 1st Semester Examination-[illegible]

MATHEMATICS

Course ID : [illegible] Course Code : [illegible]

Course Title : *Abstract Algebra*

Time : 2 Hours Full Marks : 40

The figures in the right hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Answer any *five* questions. [illegible]

1. (a) Prove that $\text{Aut}(\mathbb{Z}_n) \cong$ [illegible]. [illegible]

   (b) Find $\text{Inn}(Q_8)$. [illegible]

   (c) Find all the abelian groups of order [illegible] up to isomorphism. [illegible]

2. (a) Let G be a group and X be a G-set. Prove that $G_{gx} = gG_xg^{-1}$ for all $g \in G$ and $x \in X$ where G_x denotes the stabilizer of x. [illegible]

Turn Over

(b) Let G be a group of order $2m$, where m is an odd integer. Show that G has a normal subgroup of order m. 3

(c) Find the class equation of S_5. 3

3. (a) Find a group whose class equation is $10=1+2+2+5$. 2

(b) State and prove Sylow's third theorem. 1+5

4. (a) Let G be a non-abelian group of order p^3 (p is a prime). Prove that $|Z(G)|=p$. 2

(b) Find a group G such that $\left|\frac{G}{Z(G)}\right|=143$ 2

(c) Let G be a group of order 231. Using Sylow's theorems, show that G is cyclic. 4

5. (a) Give an example of a (i) normal series which is also a compostion series, and (ii) a normal series but not a composition series. 2+2

(b) "Every nilpotent group is abelian" – True or false ? Justify. 2

(c) State the Fundamental theorem for finitely generated abelian groups. 2

6. (a) Prove that a commutative ring with identity is simple if and only if it is a field. 2

(b) Find the ideals of the ring $\mathbb{R}[x]/<x^2-5x+4>$. 3

(c) In the ring $\mathbb{Z}[x]$, find a non-trivial prime ideal which is not maximal. 3

7. (a) Give an example of a prime element in a ring which is not irreducible. 1

(b) In a PID R, show that a non-zero non-unit element p is irreducible if and only if p is prime. 4

(c) Is Every Euclidean domain a PID ? Justify your answer. 3

8. (a) Let R be a commutative ring with identity such that $R[x]$ is a PID. Show that R is a field. 3

(b) Show that the ring $\mathbb{Q}[x]/<x^2+x+1>$ is a field. 3

(c) Show that the polynomial $2x^4+6x^3-9x^2+15$ is irreducible over $\mathbb{Z}$. 2
